3.2

Polynomial Power

Power Functions

LEARNING GOALS

In this lesson, you will:

- Determine the general behavior of the graph of even and odd degree power functions.
- Derive a general statement and explanation to describe the graph of a power function as the value of the power increases.
- Use graphs and algebraic functions to determine symmetry of even and odd functions.
- Determine whether a function is even or odd based on an algebraic function or graph.
- Understand the structure of the basic cubic function.
- Graph the basic cubic function using reference points and symmetry.

KEY TERMS

- power function
- end behavior
- symmetric about a line
- symmetric about a point
- even function
- odd function

How strong are you? Did you ever try to pick something up just to see if you could lift it? Often times, the weight a person can lift depends on that person's weight. People who weigh more tend to be able to lift more.

Powerlifting, a sport originating in the 1950's, developed separate weight classes for competitors in order to maintain a sense of fairness. Powerlifting consists of athletes competing in specific lifts: squat, bench press, and deadlift. The USA Powerlifting competition starts in high school, where young men in the 114 pound weight class are able to bench press over 250 pounds; while men in the 181 pound weight class have benched over 400 pounds. This competition is not only for men—high school women compete as well. Women in the 132 pound weight class have benched over 215 pounds.

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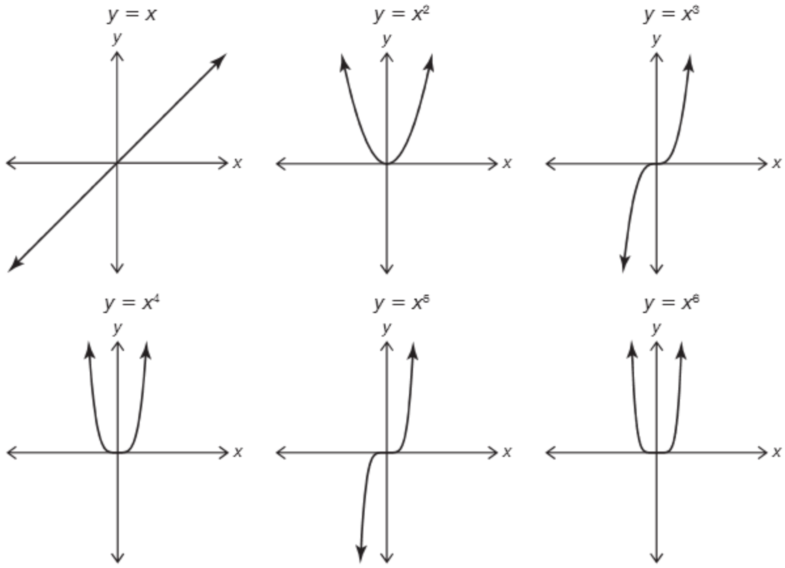
PROBLEM 1 What Odd Behavior . . . or Is It Even?

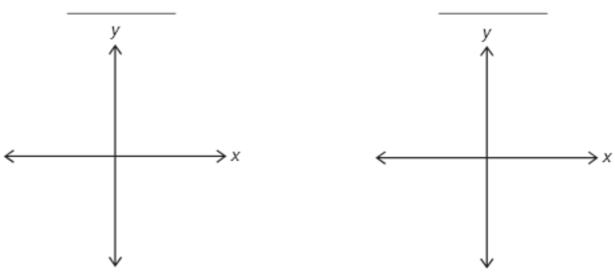
You have studied linear functions, quadratic functions, and now you will explore more polynomial functions. A common type of polynomial function is a *power function*. A **power function** is a function of the form $P(x) = ax^n$, where n is a non-negative integer.

For the purpose of this lesson, you will only focus on power functions where $a = 1$ and -1. In the next lesson you will investigate power functions with various a-values.

1. Consider each power function and its graph in the sequence shown.



a. Sketch and label the next two graphs in the sequence.



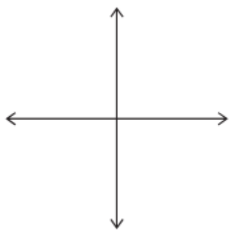
b. State any observations or patterns that you notice about the graphs in the sequence.

c. Make a general statement about the graph of a power function raised to an odd degree.

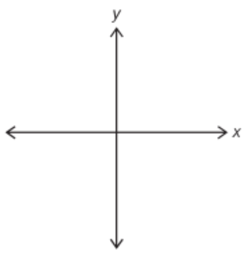
d. Make a general statement about the graph of a power function raised to an even degree.

2. Based on you work in Question 1, sketch the graph of x^n when:

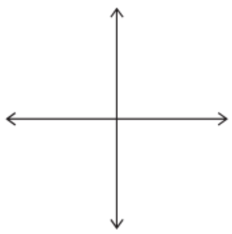
a. $n = 12$



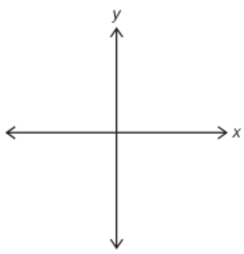
b. $n = 27$



c. $n = 2m$, where m is an integer greater than 0



d. $n = 2m + 1$, where m is an integer greater than 0

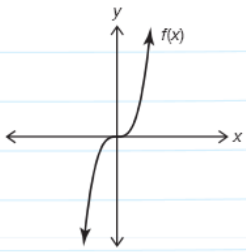


The **end behavior** of a graph of a function is the behavior of the graph as x approaches infinity and as x approaches negative infinity.

You can write the end behavior of this polynomial function using the notation:

As $x \to \infty$, $f(x) \to \infty$.

As $x \to -\infty$, $f(x) \to -\infty$.



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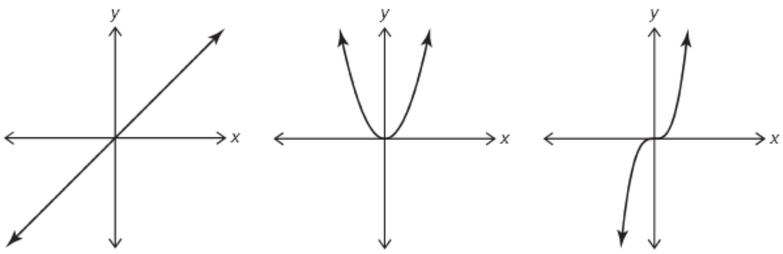
3. Explain in words what the end behavior in the worked example means.

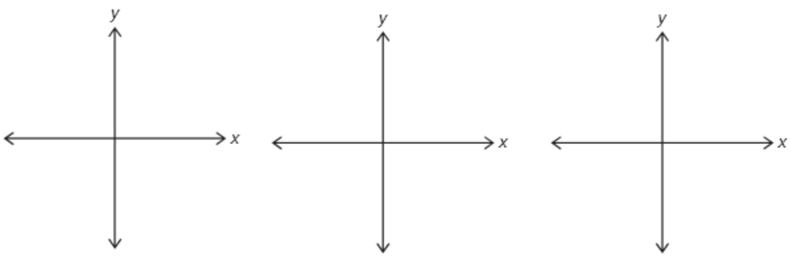
4. Consider the sequence of graphs shown.

$f_1(x) = x$ $f_2(x) = x^2$ $f_3(x) = x^3$



a. Write each function in terms of x, and then sketch it.

$-f_1(x) =$ _______ $-f_2(x) =$ _______ $-f_3(x) =$ _______



b. Complete the table to describe the end behavior for any polynomial function.

	Odd Degree Power Function	Even Degree Power Function
$a > 0$		
$a < 0$		

PROBLEM 2 If It's Flat, Then How Is It Rising?

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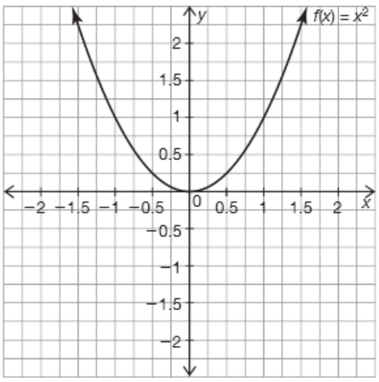
1. The function, $f(x) = x^2$, has been graphed for you. Complete the tables for $g(x) = x^4$ and $h(x) = x^6$. Then use your knowledge of the axis of symmetry to graph and label each function on the same coordinate plane shown.

x	$g(x) = x^4$
0	
0.5	
1	

x	$h(x) = x^6$
0	
0.5	
1	



a. Notice how all 3 graphs intersect at (0, 0), therefore $f(0) = g(0) = h(0) = 0$. Describe any other intersection points using function notation.

b. As the even degree power increases, what do you notice about the graph?

c. Sketch the graph of $k(x) = x^{12}$ on the same coordinate plane as $g(x)$ and $h(x)$.

d. Explain why the graphs of the even degree functions flatten as the degree increases for values of x between -1 and 1.

e. Explain why the graphs of the greater even degree functions steepen when the distance from x exceeds 1.

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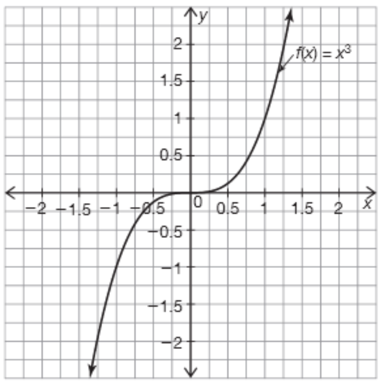
2. The function, $f(x) = x^3$, has been graphed for you. Complete the tables for $g(x) = x^5$ and $h(x) = x^7$. Then use your knowledge of the axis of symmetry to graph and label each function on the same coordinate plane.

x	$g(x) = x^5$
0	
0.5	
1	

x	$h(x) = x^7$
0	
0.5	
1	



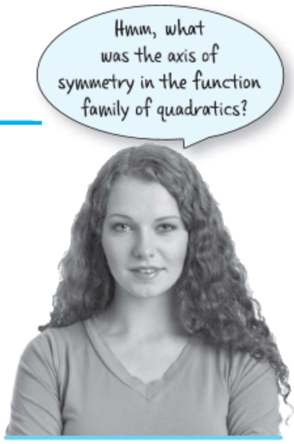
a. Notice how all 3 graphs intersect at (0, 0), therefore $f(0) = g(0) = h(0) = 0$. Describe any other intersection points using this function notation.

b. As the odd degree power increases, what do you notice about the graph?

c. Sketch and label the graph of $k(x) = x^{13}$ on the same coordinate plane as $g(x)$ and $h(x)$.

d. Explain why the graphs of the odd degree functions flatten as the degree increases for values of x between -1 and 1.

e. Explain why the graphs of the greater odd degree functions steepen when the distance from x exceeds 1.

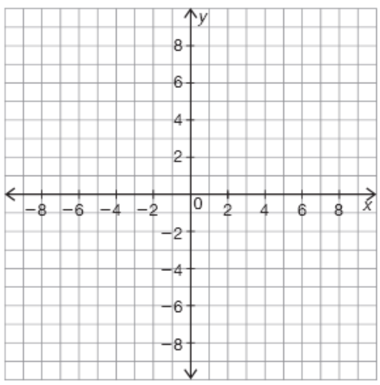


PROBLEM 3 Where's the Other Half?

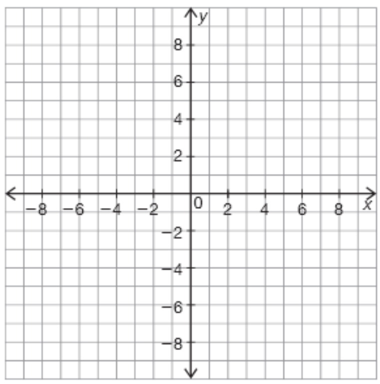
Recall that the axis of symmetry divides the graph into two parts that are mirror images of each other. If you do a reflection across an axis and the graph looks exactly the same as the original, it means that the graph is symmetric with respect to that axis.

1. Sketch 2 graphs that are symmetric to:

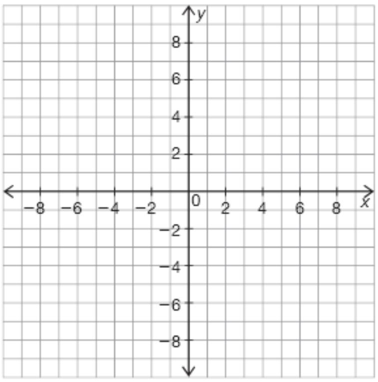
a. the x-axis



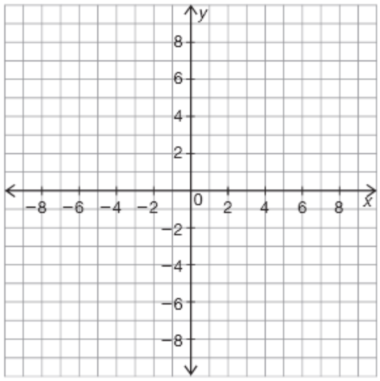
b. the line $x = 0$



c. the line $x = 3$



d. the line $y = 0$



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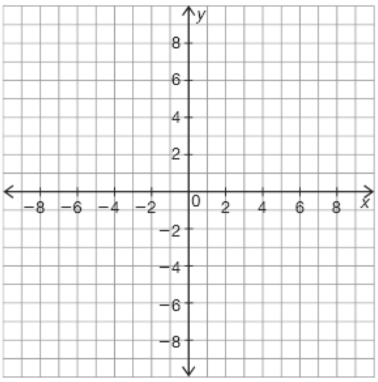
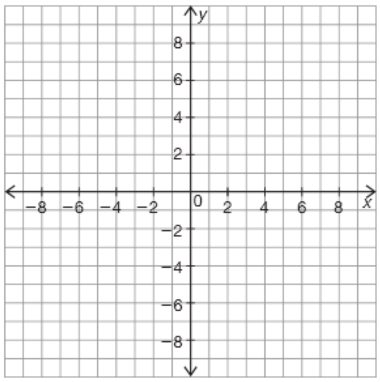
e. the line $x = -4$

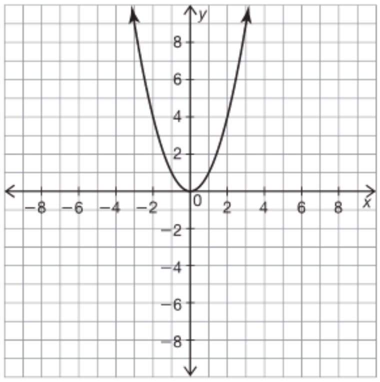


f. the line $y = 2$



g. Is each of your sketches a function? Explain why or why not.

2. Analyze the graph shown.



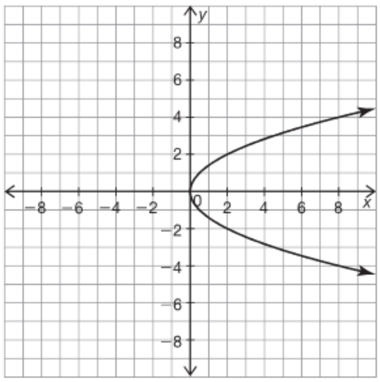
a. Identify 2 symmetric points.

b. If one point is (x, y) what are the coordinates of the other symmetric point?

c. What do you notice about the y-values when you replace x with $-x$?

d. Write a general statement to explain the relationship between any two points symmetrical to the line $x = 0$.

3. Analyze the graph shown.



a. Identify 2 symmetric points.

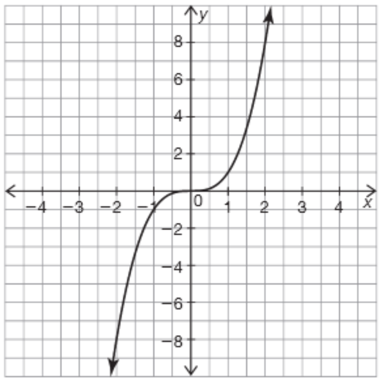
b. If one point is (x, y) what are the coordinates of the other symmetric point?

c. What do you notice about the x-values when you replace y with $-y$?

d. Write a general statement to explain the relationship between any two points symmetrical to the line $y = 0$.

If a graph is **symmetric about a line**, the line divides the graph into two identical parts. Special attention is given to the line of symmetry when it is the *y*-axis as it tells you that the function is even.

4. Analyze the graph shown.



Olivia says that the graph has no line of symmetry because if she reflected the graph across the *x*- or *y*-axis, it would just change the graph to look like an odd degree power function with a negative *a*-value, thus not looking like a mirror image.

Randall says that the graph has no line of symmetry because if he looks at the *x*-value at 1 and −1, the *y*-value is not the same, so there can't be symmetry about the *y*-axis. Also if he looks at the *y*-value at 8 and −8, the *x*-value is not the same, so there can't be symmetry about the *x*-axis.

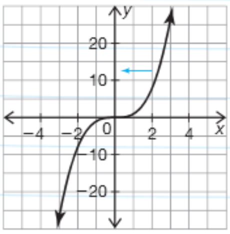
Shedrick said that there is some type of symmetry. He notices that if he looks at the point (2, 8) the point (−2, −8) is also on the graph. Likewise he looks at the point (1, 1) and notices that the point (−1, −1) is also on the graph. He concluded that it must have a reflection across the *x*- and *y*-axis at the same time.

Who's correct? Explain your reasoning.

The graph of an odd degree basic power function is *symmetric about a point*, in particular the origin. A function is **symmetric about a point** if each point on the graph has a point the same distance from the central point, but in the opposite direction. Special attention is given when the central point is the origin as it determines that the function is odd. When the point of symmetry is the origin, the graph is reflected across the x-axis and the y-axis. If you replace both (x, y) with $(-x, -y)$, the function remains the same.

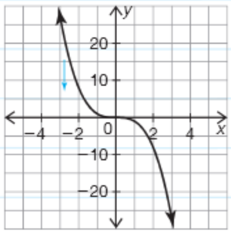
You can think of the point of symmetry about the origin, as a double reflection.

$f_1(x) = x^3$



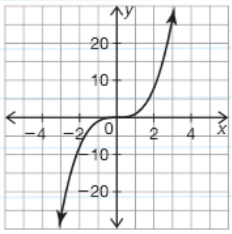
The function $f_1(x)$ is shown.

$f_2(x) = f_1(-x)^3$
$= (-x)^3$



The function $f_1(x)$ is reflected across the y-axis to produce f_2.

$f_3(x) = -f_2(x)$
$= x^3$



The function $f_2(x)$ is reflected across the x-axis to produce f_3.

An **even function** has a graph symmetric about the y-axis, thus $f(x) = f(-x)$.

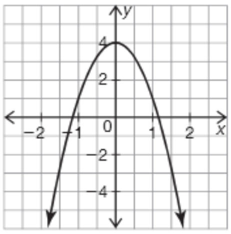
An **odd function** has a graph symmetric about the origin, thus $f(x) = -f(-x)$.

5. Which graph in Questions 2 through 4 represents an even function? Explain your reasoning.

6. Which graph in Questions 2 through 4 represents an odd function? Explain your reasoning.

7. State whether the graph of each function shown is even, odd, or neither.

a.



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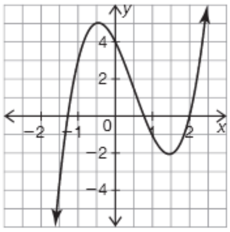
b.



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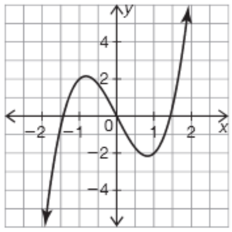
c.



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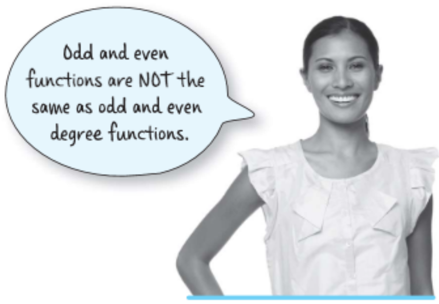

8. Lillian and Destiny are working on the problem shown.

Determine algebraically whether the polynomial function $f(x) = 3x^4 - 2x^3 + 4x - 6$ is even, odd, or neither.

Lillian

$f(x) = 3x^4 - 2x^3 + 4x - 6$

$f(-x) = 3x^4 - 2x^3 + 4x - 6$
$f(-x) = 3(-x)^4 - 2(-x)^3 + 4(-x) - 6$
$f(-x) = 3x^4 + 2x^3 - 4x - 6$

$-f(x) = -(3x^4 - 2x^3 + 4x - 6)$
$-f(x) = -3x^4 + 2x^3 - 4x + 6$

$f(x) \neq f(-x)$ or $-f(x)$ thus
$f(x)$ is neither even or odd.

Destiny

$f(x) = 3x^4 - 2x^3 + 4x - 6$

$f(-x) = 3x^4 - 2x^3 + 4x - 6$
$f(-x) = 3(-x^4) - 2(-x^3) + 4(-x) - 6$
$f(-x) = -3x^4 + 2x^3 - 4x - 6$

$-f(x) = -(3x^4 - 2x^3 + 4x - 6)$
$-f(x) = -3x^4 + 2x^3 - 4x + 6$

$f(x) \neq f(-x)$ or $-f(x)$ thus
$f(x)$ is neither even or odd.

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a. Explain why Destiny's work is incorrect.

b. How can you use algebra to determine whether a function is even or odd?

9. Determine algebraically whether the functions are even, odd, or neither.

a. $f(x) = 2x^3 - 3x$

b. $g(x) = 6x^2 + 10$

c. $h(x) = x^3 - 3x^2 - 2x + 7$

Be prepared to share your solutions and methods.